

Discrete Quantum Optics

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1. Show that the eigenvectors of a linear operator \hat{A} are linearly independent.
 2. Show that $(\hat{A} + \hat{B})^\dagger = \hat{A}^\dagger + \hat{B}^\dagger$, and $(\hat{A}\hat{B})^\dagger = \hat{B}^\dagger\hat{A}^\dagger$.
 3. Show that a projector operator \hat{P} is Hermitian and can be inverted if and only if $\hat{P} = I$, where I is the identity operator.
 4. Show that the standard Pauli matrices σ_x , σ_y , and σ_z are Hermitian and unitary.
 5. Let be $|+\rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ 1 \end{pmatrix}$ and $|-\rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ -1 \end{pmatrix}$, write down the Pauli matrices in the basis $\{|+\rangle, |-\rangle\}$.
 6. Show that if the operators \hat{A} and \hat{B} are Hermitian, then $i[\hat{A}, \hat{B}]$ is also Hermitian.
 7. Show that any unitary operator \hat{U} can be written as $\hat{U} = \exp(i\hat{A})$, where \hat{A} is a Hermitian operator.
 8. Consider the Hamiltonian $\hat{H} = \begin{pmatrix} \beta & \kappa \\ \kappa & \beta \end{pmatrix}$, compute the evolution operator $\hat{U}(t) = \exp(-it\hat{H})$ using (i) the spectral decomposition and (ii) expanding the exponential in Taylor series.
 9. Given the operator momentum operator $\hat{p} = -i\frac{\partial}{\partial x}$, show that the commutator $[\hat{x}, \hat{p}] = i$.
 10. Show the identities:
 - $\exp\left(\frac{it\hat{p}^2}{2}\right) h(\hat{x}) \exp\left(\frac{-it\hat{p}^2}{2}\right) = h(\hat{x} + t\hat{p})$
 - $\exp\left(\frac{iu(\hat{x}+t\hat{p})}{2}\right) = \exp\left(\frac{it u^2}{2}\right) \exp(iu\hat{x}) \exp(itu\hat{p})$
 - $\exp\left(\frac{it\hat{p}^2}{2}\right) \exp(iv\hat{x}) = \exp\left(\frac{itv^2}{2}\right) \exp(iv\hat{x})$
 - $\exp(itu\hat{p}) \exp(iv\hat{x}) = \exp(ituv) \exp(iv\hat{x})$
 - $\exp\left(\frac{it\hat{p}^2}{2}\right) \exp\left(\frac{-it\hat{x}^2}{2\sigma^2}\right) \exp\left(\frac{-it\hat{p}^2}{2}\right) = \exp\left[\frac{-(\hat{x}^2 + (\hat{x}\hat{p} + \hat{p}\hat{x}) + t^2\hat{p}^2)}{2\sigma^2}\right]$
 11. Consider the spectral decomposition of the Fourier operator $\hat{F}_\alpha = \sum_{n=0}^{\infty} |\varphi_n\rangle \langle \varphi_n| \exp(in\alpha)$, where $|\varphi_n\rangle$ represents a normalized Hermite-Gauss polynomials of order n . Using matlab or mathematica, maple, etc, to apply such a Fourier operator with $\alpha = \frac{\pi}{2}, \frac{\pi}{4}$ to a rect function defined as $\Pi(x) = \begin{cases} 0 & \text{if } |x| > 1 \\ 1 & \text{if } |x| = 1 \\ 0 & \text{if } |x| < 1 \end{cases}$.
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