Fluctuation-induced Phenomena: Problem Set 0

1) Consider the wave function $\psi(x, t)$ defined as

$$\psi(x, t) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{+\infty} dk \tilde{\psi}(k) e^{i(kx-\omega(k)t)}$$

– How do I calculate $\tilde{\psi}(k)$?
– What is the behavior of $\tilde{\psi}(k)$ if

$$\psi(x; t = 0) = e^{-\frac{(x-x_0)^2}{4\sigma^2}} e^{\frac{i}{\hbar}p_0 x} \quad ?$$

2) Consider the Hamiltonian

$$\hat{H}_0 = \hbar \omega_0 \left( \hat{a}^\dagger \hat{a} + \frac{1}{2} \right)$$

– Which system do I describe with it?
– How are the eigenstates called?
– What is the value of $[\hat{a}, \hat{a}^\dagger]$?

3) Consider the operator

$$\hat{U}(t) = e^{-\frac{\hbar}{i} \hat{H}_0 t}$$

– What is its definition?
– What is the result of

$$e^{\frac{\hbar}{i} \hat{H}_0 t} \hat{a} e^{-\frac{\hbar}{i} \hat{H}_0 t} = ?$$

4) Consider the equation

$$\ddot{x}(t) + \gamma \dot{x}(t) + \omega_0^2 x(t) = f(t)$$

– Which system do I describe with it?
– What is $\gamma$?
5) Consider the equation

$$\nabla \times \nabla \times E(r, t) + \frac{\partial^2}{\partial t^2} \int_{-\infty}^{\infty} dt' \epsilon(t - t')E(r, t') = -\mu_0 \frac{\partial}{\partial t} j(r, t)$$  \hspace{1cm} (7)

- Which system do I describe with it?
- What is $\epsilon(t - t')$?
- What are the properties of the Fourier transfer of $\epsilon(t - t')$, $\epsilon(\omega)$ for $\omega \to \infty$ and $\omega \to 0$?

- Which kind of dynamics should I expect?