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Fluctuation-induced Phenomena: Problem Set 4

1) Casimir oscillator

Consider the set-up in Figure 1. The spring constant K is known, as is the mass M of each of the two metallic plates. At equilibrium, the distance between the two plates is d. The left plate is now displaced from the equilibrium by a small distance, causing the system to oscillate.

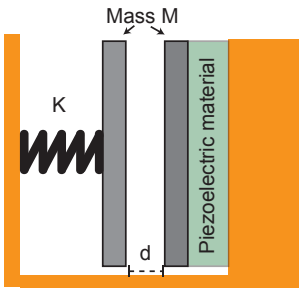

Figure 1: Set up of a Casimir oscillator consisting of two plates of area A and mass M separated by a distance d. One plate is attached to a spring with spring constant K, the other is attached to a piezoelectric material to provide precise control of the distance.

(a) Show that the Casimir force between the plates leads to an oscillation frequency which depends on the distance d (as well as the area of the plates). Calculate the frequency shift with respect to the oscillator's eigenfrequency.

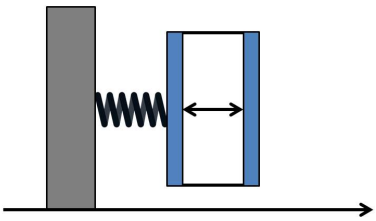

Figure 2: Set up of a Casimir oscillator the two plates of area A and mass M are connected (at the edges) and disconnected from the piezoelectric material. The cavity is still attached to a spring with spring constant K.

(b) As a thought experiment, suppose now that the whole cavity is oscillating (i.e. the two plates are connected (at the edges) and disconnected from the piezoelectric material, see Fig. 2). Use the mass-energy equivalence to show that also in this case the frequency is expected to depend on the distance between the plates.

2) The central limit theorem. In physics the Gaussian distribution is omnipresent and it plays a leading role in the theory of stochastic process. Mathematically, this predominance can be partially understood through the central limit theorem. It says that, given a set of n random variables $X_1, X_2, \ldots X_n$ each of them characterized by an independent probability distribution $P_i(x_i)$ (not necessarily identical) with zero average and an the same finite

variance σ^2, the distribution $P_n(z)$ of the random variable

$$Z = \frac{1}{\sqrt{n}} \sum_{i=1}^{n} X_i, \tag{1}$$

is Gaussian in the limit $n \to \infty$. The demonstration of this simple but rather profound result was only sketched in the script, and relies on the use of the characteristic function. Repeat and complete the demonstration.

2) Properties of the Ornstein-Uhlenbeck stochastic process. The Ornstein-Uhlenbeck stochastic process is at same the time, stationary, Markovian, and also Gaussian. One of the characteristic of this process is that the second-order correlation has the property

$$C_2(t_3 - t_1) = C_2(t_3 - t_2)C_2(t_2 - t_1). \tag{2}$$

Show that the only function that satisfies this property is an exponential.
(Hint: A possible way it to get a differential equation for $C_2(\tau)$ using a Taylor expansion.)