

Discrete Quantum Optics

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1. Two coupled oscillators are described by the hamiltonian

$$H = \omega_1 \hat{a}^\dagger \hat{a} + \omega_2 \hat{b}^\dagger \hat{b} + \kappa (\hat{a}^\dagger \hat{b} + \hat{a} \hat{b}^\dagger).$$

- Consider $\omega_1 = \omega_2 = \omega$ and solve the Heisenberg equations of motion for $\hat{a}^\dagger(t)$ and $\hat{b}^\dagger(t)$.
- Compute the expressions $\hat{a}_H^\dagger(t) = U^\dagger(t) \hat{a}_S^\dagger U(t)$ and $\hat{b}_H^\dagger(t) = U^\dagger(t) \hat{b}_S^\dagger U(t)$, where the subscripts H and S designate the Heisenberg and the Schrödinger pictures, respectively. Compare your results with the solutions of the corresponding Heisenberg equations.
- Think about the case when $\omega_1 \neq \omega_2$, compute the eigenvectors and eigenvalues for the hamiltonian and use the spectral decomposition to find the evolution operator.
- Write a Matlab script to compute the solution using the Runge-Kutta method. Compare the analytical and numerical results.