Humboldt–Universität zu Berlin
Institut für Physik
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SoSe 2017
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http://www.physik.hu-berlin.de/top/teaching

Computational Photonics: Problem Set 5

1 1 Nonlinear Finite-Difference Time-Domain

a) Instantaneous Kerr-nonlinearity

Consider a one-dimensional homogeneous system which is filled with an instantaneous Kerr-nonlinear medium so that the nonlinear polarization is

$$P_{\mathrm{NL}}(x) = \chi_0^{(3)} E^3.$$

Write down Maxwell's equations of this system with the electric field E, the magnetic field H and the dielectric displacement field D. Together with the constitutive (material) equations, we obtain a system of three coupled equations which we analyze in the remainder of the problem.

b) Discretization

Discretize the three equations on staggered grids analogous to the Finite-Difference Time-Domain approach in Problem Set 2. In the present case you also have to discretize the dielectric displacement field. This is done on the same grid as that used for the electric field. Rewrite the resulting equations as an update algorithm, i.e., obtain the values for the fields at a given time step using only the fields at earlier time steps.

c) Implementation of a 1D nonlinear FDTD code

Implement the discretized equations in a system with metallic boundaries (PEC or PMC). Use the same technique as discussed in Problem Set 2. The initial condition for the electric field shall be a Gaussian-shaped pulse that, at $t = 0$, is located in the middle of the system. In order to be able to execute the first step in time for the electric field, you also need to initialize the magnetic field and the dielectric displacement field at different times $t \neq 0$. Determine these initial fields analytically from the initial condition of the electric field with the assumption that the medium is linear. Remember that electric and magnetic field are staggered in time and space!

The discretized constitutive equation is a cubic equation and can be solved numerically, e.g., through Newton iteration

$$f(x) = 0 \quad \Rightarrow \quad x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}.$$

d) Pulse-steepening

Simulate the propagation of a Gaussian pulse in a medium with a Kerr nonlinear coefficient $\chi_0^{(3)} = 0.1$ and a linear dielectric constant $\epsilon_{\text{linear}} = 1$. Due to the Kerr nonlinearity the pulse will steepen and finally break. In order to observe the effect clearly, your (discretized) pulse must not be too narrow.

e) Third-harmonic generation

Simulate a Gaussian pulse

$$E(x, t = 0) = \exp(-\frac{x^2}{2a_0^2}) \cos(k_c x),$$

with width a_0 and a carrier wavenumber $k_c \neq 0$. Monitor the changes in the spectrum of the pulse: If you plot, after each time step, the Fourier transformed pulse, you will observe that - relative to the spectrum at $t = 0$ - new frequency components are generated by the Kerr nonlinearity.

f) Sources of errors

When the simulation starts you can observe a small apparent disturbance propagating in the opposite direction away from the pulse. Can you imagine the reason for this unphysical behavior?

— Discussion in the exercise on July 21st, 2017 —

Problems marked with * are voluntary. All others are **due** on **July 21st, 2017** at **8am** via e-mail to kurt.busch@physik.hu-berlin.de or have to be brought to the exercise class on a flash drive. Analytical solutions are accepted handwritten or in the .pdf format, numerical solution in a digital version.