Humboldt–Universität zu Berlin SoSe 2018
Institut für Physik
Prof. Dr. Kurt Busch, Bettina Beverungen May 22th, 2018
https://top.physik.hu-berlin.de/teaching

Computational Photonics: Problem Set 4

1 Free-space BPM

a) Semi-circle free-space BPM operator

Consider a beam that propagates in x–direction and is independent of z. Determine the propagator U_{FS} of the free-space system. The propagator U_{FS} evolves the system from point x to point $x + \Delta x$, i.e. $E_z(x + \Delta x, y) = U_{\mathrm{FS}} E_z(x, y)$. Note: Fourier transform the equation with respect to the y-component.

b) Implementation

Implement the classical free-space BPM. Use as initial pulse a transverse (i.e., in y-direction) Gaussian profile and plot the electric field distribution in the xy-plane after completing the simulation. Check the influence of the system's width in y-direction and the step width in x-direction. What happens if the pulse approaches the boundaries?

2 BPM with planar waveguide

a) Waveguide BPM operator

Simulate a planar waveguide (see figure 1) with the beam propagation method. Use a (transverse) step index profile for the dielectric constant $\epsilon(y)$. The operator H_{TE} can be split into two parts $H_{\mathrm{TE}} = D + W$. D is the so-called diffraction operator and W contains all inhomogeneities of the dielectric profile. The dielectric profile can be written as $\epsilon(y) = \bar{\epsilon} + \Delta\epsilon(y)$, where the reference dielectric constant $\bar{\epsilon}$ is included in D and $\Delta\epsilon(y)$ is part of W. Assume that W is a small perturbation to the free propagation described by D and expand the operator $\sqrt{H}$ of the forward Helmholtz equation into powers of W. Make a paraxial approximation to $\sqrt{D}$. The final propagator U can then be calculated by using the operator expansion $\exp(A + B) \approx \exp(A/2)\exp(B)\exp(A/2)$. Interpretation: To obtain one step in space Δx, we first propagate half the step size in homogeneous space, then add a phase correction to include the small perturbation by $\Delta\epsilon$ and finally propagate the remaining half step size in homogeneous space.

b) Implementation

Implement the classical BPM for a planar waveguide with thickness d_{wg}. Remember to choose the reference dielectric constant such that the condition $\|\Delta\epsilon(y)E_z(x,y)\|_{\max} \ll \|\bar{\epsilon}E_z(x,y)\|_{\max}$ is satisfied. Again, use a Gaussian pulse as initial condition and simulate its evolution in the planar waveguide. Check the influence of the step size.

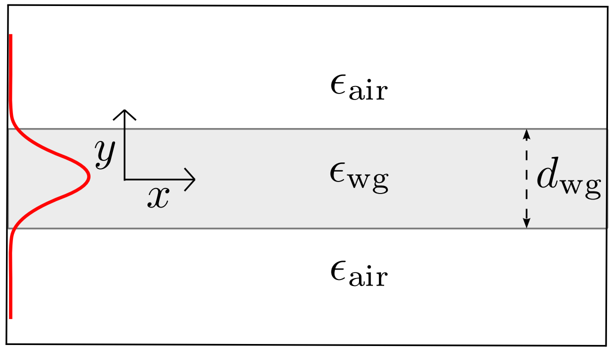

Figure 1: Planar waveguide

3 Split-step Fourier method

The nonlinear Schrödinger equation (NLSE) describes the propagation of pulses in dispersive media that exhibit a nonlinear index of refraction

$$i\frac{\partial}{\partial t}a(z,t) + \frac{\omega_2}{2}\frac{\partial^2}{\partial z^2}a(z,t) + \chi^{(3)}\left|a(z,t)\right|^2 a(z,t) = 0,$$

where $a(z,t)$ is the slowly varying evelope of a pulse considered in a frame that moves with the group velocity of the pulse. In the moving frame, the complete pulse is given by $A(z,t) = a(z,t)\exp(ik_0 z - i\omega_0 t)$ with a carrier wave that is characterized by a wave number k_0 and the associated carrier frequency ω_0. Finally, $\chi^{(3)}$ denotes the nonlinear susceptibility and ω_2 is an abbreviation for the group velocity dispersion at frequency ω_0, i.e., $\omega_2 = \partial^2\omega(k)/\partial k^2|_{k=k_0}$.

a) The NLSE can be solved numerically through the split-step Fourier method. To this end, the equation is split into a linear and a nonlinear part $\frac{\partial}{\partial t}a(z,t) = (D+N)a(z,t)$. To solve this equation for small time steps Δt, we use a technique similar to the beam propagation method discussed in Problem Set 2: First, a purely linear time step of length $\Delta t/2$ is calculated in Fourier space, then the compounded effect of the nonlinear part over the entire time step Δt is applied in real space and finally the second half of the time step is again executed as a purely linear step in Fourier space. The integral that appears in the nonlinear exponential operator is solved in an iterative fashion in order to improve the accuracy: First, the integral is approximated by the value of the integrand at the lower boundary ($N(t_0)$). Through time-stepping this results in an approximation for the integrand at the upper boundary ($N(t_0+\Delta t)$). Then, the same time step is repeated using the trapezoidal rule for the integral. Write down the equations to advance your system one step in time.

b) Implement the symmetrized split-step Fourier method and test it with an initial Gaussian shaped pulse. The nonlinear susceptibility $\chi^{(3)}$ is assumed to be positive, but the sign of the group velocity dispersion ω_2 can be negative or positive. If you multiply the phase factor with the carrier wave number to your calculated slowly varying amplitude $(a(z,t)e^{ik_0 z})$ you can observe a down-chirp in your pulse with the positive dispersion and an up-chirp with the negative dispersion.

The NLSE has soliton solutions

$$a(z,t) = \operatorname{sech}\left(\sqrt{\frac{\chi^{(3)}}{\omega_2}}z\right)\exp\left(i\chi^{(3)}\frac{t}{2}\right).$$

Test the method also with this pulse shape.

— Discussion in the exercise on June 8th, 2018 —

Problems marked with * are voluntary. All others are **due** on **June 8th, 2018** at **8am** via e-mail to bettina.beverungen@physik.hu-berlin.de or have to be brought to the exercise class on a flash drive. Analytical solutions are accepted handwritten or in the .pdf format, numerical solution in a digital version.