Humboldt–Universität zu Berlin
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Fluctuation-induced Phenomena: Problem Set 10

1) Excitation rates (10 Points). Perturbation theory and in particular Fermi's golden rule allows us to calculate the transition rates. Consider, for instance, a two-level atom in its ground state $|g\rangle$ surrounded by bodies and by the thermal radiation. If the atom is excited to its excited state $|e\rangle$, the corresponding transition rate γ_- of the process reads

$$\gamma_- = \frac{2\pi}{\hbar} \sum_I p_I \sum_F \left| \langle F|\langle e|\hat{\mathbf{d}} \cdot \hat{\mathbf{E}}(\mathbf{r}_0)|g\rangle|I\rangle \right|^2 \delta(E_F + E_e - E_I - E_g) \ . \tag{1}$$

Here, $p_I = e^{-\beta E_I}/\sum_k e^{-\beta E_k}$, $\beta = [k_B T]^{-1}$ and the dipole operator is $\hat{\mathbf{d}} = \mathbf{d}(|e\rangle\langle g| + |g\rangle\langle e|)$. The state $|I\rangle$ and $|F\rangle$ are eigenstates of the *free* (without the atom) field Hamiltonian $\hat{H}_{\text{field}}$ with eigenenergies E_I and E_F, respectively. The states $|g\rangle$ and $|e\rangle$ are the eigenstates of the dipole Hamiltonian with eigenenergies E_g and E_e, respectively.

(a) Show that we can write

$$\gamma_- = \frac{1}{\hbar^2} \int_{-\infty}^{\infty} d\tau \, \mathbf{d} \cdot \text{tr}\left[\hat{\mathbf{E}}^{\text{free}}(\mathbf{r}_0, \tau) \hat{\mathbf{E}}^{\text{free}}(\mathbf{r}_0, 0) \hat{\rho}_{\text{field}} \right] \cdot \mathbf{d} \, e^{-\mathrm{i}\omega_0 \tau} \ , \tag{2}$$

where $\omega_0 = (E_e - E_g)/\hbar$ (dipole transition frequency), $\hat{\rho}_{\text{field}} = Z^{-1} e^{-\beta \hat{H}_{\text{field}}}$ and $Z = \text{tr}[e^{-\beta \hat{H}_{\text{field}}}]$.

(b) Simplify the previous expression using the fluctuation-dissipation theorem

$$\langle \hat{\mathbf{E}}^{\text{free}}(\mathbf{r}, \omega) \hat{\mathbf{E}}^{\text{free}}(\mathbf{r}', \omega') \rangle = \frac{4\pi\hbar}{1 - e^{-\beta\hbar\omega}} \frac{Z_0}{c} \omega^2 \text{Im}\left[\underline{G}(\mathbf{r}, \mathbf{r}'; \omega) \right] \delta(\omega + \omega') \tag{3}$$

where $\underline{G}(\mathbf{r}, \mathbf{r}'; \omega)$ is the electric Green tensor, Z_0 is the vacuum impedance and c is the speed of light.

(c) Conclude that the process $|g\rangle \to |e\rangle$ occurs only for $T > 0$ (as it should).

— 10 Points —
— To be handed in prior to the tutorials on 03.07.2018 —