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Fluctuation-induced Phenomena: Problem Set 10

1) Atomic polarizability. One of the recurring concepts in the description of atoms interacting with the electromagnetic field is its linear response tensor

$$\underline{\alpha}(\tau) = \frac{\mathrm{i}}{\hbar}\theta(\tau)\langle[\hat{\mathbf{d}}(\tau), \hat{\mathbf{d}}(0)]\rangle \ , \tag{1}$$

where $\theta(\tau)$ is the Heaviside function and the mean value is taken over the state of interest. For the response function one requires the time evolution only with respect to the free atomic Hamiltonian characterized by the eigenstates

$$\hat{H}_{\text{atom}}|E_n\rangle = E_n|E_n\rangle \ . \tag{2}$$

(a) Assuming that the dipole operator has vanishing diagonal elements, show that if the state of interest is $|E_a\rangle$ we have that the polarizability (the Fourier transform of the atomic response function) is given by

$$\alpha^a_{ij}(\omega) = \sum_n \frac{d_i^{an} d_j^{na}}{\hbar} \frac{2\omega_{na}}{\omega_{na}^2 - (\omega + \mathrm{i}\epsilon)^2} \tag{3}$$

where $d_j^{na} = \langle E_n|\hat{d}_j|E_a\rangle$ is the dipole matrix element, $\omega_{na} = (E_n - E_a)/\hbar$ and ϵ has a vanishing small value. Comment on the sign of ϵ.

(b) Which state should one consider to obtain the atomic polarizability for an atom in thermal equilibrium at temperature T?

(c) Calculate the thermal polarizability and show that

$$\alpha^T_{ij}(\omega) = \sum_a \frac{e^{-E_a/k_B T}}{Z} \alpha^a_{ij}(\omega) \tag{4}$$

where Z is a scalar (give its expression).

(d) Show that in the limit $T \to 0$, $\alpha^T_{ij}(\omega)$ becomes the polarizability for the atomic ground state.