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Fluctuation-induced Phenomena: Problem Set 8

1) A quantum system of charges interacting with the electromagnetic field. A system like an atom interacting with the classical electromagnetic field in the Coulomb gauge is described by the Hamiltonian

$$\hat{H}(t) = \sum_i \frac{1}{2m_i} [\hat{\mathbf{p}}_i - q_i \mathbf{A}(\mathbf{r}_i, t)]^2 + \frac{1}{8\pi\epsilon_0} \sum_{i \neq j} \frac{q_i q_j}{|\hat{\mathbf{r}}_i - \hat{\mathbf{r}}_j|} \tag{1}$$

where $\mathbf{A}(\mathbf{r}, t)$ is the vector potential, q_i are the charges and ϵ_0 is the vacuum permittivity (we have neglected here the Coulomb selfenergy). If the size of the system is smaller than the wavelength of the radiation we can neglect the spatial variation of the radiation. We can then set in the vector potential all the $\mathbf{r}_i$ to the same value which coincide with a point $\mathbf{R}$ in the interior of the system of charges. We will also assume that $\mathbf{R} = 0$, i.e. the origin of our frame.

(a) Consider the unitary transformation $\hat{T}(t)$. Shows that the evolution of the state $|\phi(t)\rangle = \hat{T}(t)|\psi(t)\rangle$ is governed by the Hamiltonian

$$\hat{H}'(t) = \hat{T}(t)\hat{H}(t)\hat{T}^\dagger(t) + \mathrm{i}\hbar \left(\frac{d\hat{T}(t)}{dt} \right) \hat{T}^\dagger(t). \tag{2}$$

(b) Consider now the transformation

$$\hat{T}(t) = \exp\left[-\frac{\mathrm{i}}{\hbar} \sum_i q_i \hat{\mathbf{r}}_i \cdot \mathbf{A}(0, t) \right] \tag{3}$$

and show that the transformed Hamiltonian is given by

$$\hat{H}'(t) = \sum_i \frac{\hat{\mathbf{p}}_i^2}{2m_i} + \frac{1}{8\pi\epsilon_0} \sum_{i \neq j} \frac{q_i q_j}{|\hat{\mathbf{r}}_i - \hat{\mathbf{r}}_j|} - \hat{\mathbf{d}} \cdot \mathbf{E}(0, t) \tag{4}$$

where $\hat{\mathbf{d}} = \sum_i q_i \hat{\mathbf{r}}_i$.