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Fluctuation-induced Phenomena: Problem Set 5

1) Brownian Motion. Consider the stochastic differential equation

$$\dot{V}(t) = -\gamma V(t) + F(t) \quad \text{with} \quad \langle F(t) \rangle = 0 \quad \text{and} \quad \langle F(t)F(t') \rangle = \Delta\, \delta(t-t'). \tag{1}$$

1. Show that

$$\langle V(t)V(t') \rangle = \left(v_0^2 - \frac{\Delta}{2\gamma} \right) e^{-\gamma(t+t')} + \frac{\Delta}{2\gamma} e^{-\gamma|t-t'|}, \tag{2}$$

   where we have assumed for simplicity that all realizations are such that $v_0 = V(t=0)$.
   Hint: Pay attention at the absolute value in the exponent.

2. Consider now the stochastic differential equation obtained by setting $V(t) = \dot{X}(t)$. Calculate $\langle X(t) \rangle$ and $\langle X(t)X(t') \rangle$.
   Hint: First find an expression for $X(t)$ by integrating the differential equation. The resulting two-dimensional integral simplifies substantially by changing the order of integration.

2) The Fokker-Planck equation. For a certain class of stochastic processes, the first order probability distribution $W_1(x,t)$ obeys the differential equation

$$\partial_t W_1(x,t) = \gamma \partial_x \left[x W_1(x,t) \right] + \frac{D}{2} \partial_x^2 W_1(x,t), \tag{3}$$

derive the equations of motion for $\langle X(t) \rangle$ and $\langle X^2(t) \rangle$.