Humboldt–Universität zu Berlin
Institut für Physik SoSe 2017

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Fluctuation-induced Phenomena: Problem Set 10

1) Casimir energy and argument principle (7 Points).

At zero temperature the Lifshitz' formula for the Casimir energy between perfectly reflecting mirrors is given by

$$E_{\mathrm{Lif}}(L) = A \int_0^\infty \frac{d\xi}{2\pi} \int \frac{\mathrm{d}^2\mathbf{k}}{(2\pi)^2} \sum_\sigma \hbar \ln\left(1 - e^{-2\kappa L}\right), \tag{1}$$

where $\kappa = \sqrt{|k|^2 + \xi^2/c^2}$. Remark that we can write

$$1 - e^{-2\kappa L} = \frac{D(\kappa, L)}{D(\kappa, L \to \infty)}, \tag{2}$$

where $D(\kappa, L) = \sinh[\kappa L]$ and $D(\kappa, L \to \infty)$ is the *asymptotic expression* of $D(\kappa, L)$ in the limit of $L \to \infty$.

Using the argument principle, show that we can recover Casimir's formula for the energy as sum over the zero point energies of the frequency modes vibrating between the mirrors.

$$E_{\mathrm{Lif}}(L) = A \int \frac{\mathrm{d}^2\mathbf{k}}{(2\pi)^2} \sum_\sigma \left[\sum_n \frac{\hbar\omega_n^\sigma(\mathbf{k}, L)}{2} \right]_{L\to\infty}^{L} \tag{3}$$

where

$$\omega_n^\sigma(\mathbf{k}, L) = c\sqrt{|\mathbf{k}|^2 + \left(\frac{\pi}{L} n\right)^2}. \tag{4}$$

Hint: Look for the zeros of $D(\kappa, L)$ in the complex-frequency plane.

— 7 Points —

— To be handed in prior to the tutorials on 17.07.2017 —