HUMBOLDT–UNIVERSITÄT ZU BERLIN SoSe 2017
INSTITUT FÜR PHYSIK

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Fluctuation-induced Phenomena: Problem Set 6

1) Modes and dissipation (10 Points). Consider a system of coupled harmonic oscillators described by the Hamiltonian

$$\hat{H} = \frac{\hat{p}^2}{2M} + \frac{1}{2}M\omega_0^2\hat{x}^2 + \sum_{n=1} \frac{1}{2m_n}\left(\hat{p}_n^2 + m_n^2\omega_n^2[\hat{q}_n - \hat{x}]^2\right). \tag{1}$$

The oscillator with mass M and frequency ω_0 is our system, while the oscillators with mass m_n and frequency ω_n describe the environment.

(a) Show that the previous Hamiltonian leads to the following equation of motions

$$\ddot{\hat{x}}(t) + \omega_0^2\hat{x}(t) = \sum_{n=1}\frac{m_n}{M}\omega_n^2[\hat{q}_n - \hat{x}], \quad \ddot{\hat{q}}_n(t) + \omega_n^2\hat{q}_n(t) = \omega_n^2\hat{x}(t) \tag{2}$$

(b) Go in the Fourier space and show that combining the equations one obtains

$$\left[-\omega^2 + \omega_0^2 - \omega^2\sum_{n=1}\frac{m_n}{M}\frac{\omega_n^2}{\omega_n^2 - \omega^2}\right]\hat{x}(\omega) = 0 \tag{3}$$

(c) If we now consider the case where the frequencies ω_n become dense to form a continuum, one can write

$$\alpha^{-1}(\omega) = -\omega^2 + \omega_0^2 - \imath\omega\mu(\omega) \quad \mu(\omega) = -\imath\omega\int_0^\infty \mathrm{d}\nu\rho(\nu)\frac{m(\nu)}{M}\frac{\nu^2}{\nu^2 - \omega^2}. \tag{4}$$

Using $\rho(\nu)\frac{m(\nu)}{M}\nu^2 = \frac{2}{\pi}\gamma$ evaluate the integral in $\mu(\omega + \imath\epsilon)$ and consider $\epsilon \to 0$ at the end of the calculation. Discuss the implications of the sign of ϵ.

2) Driven harmonic oscillator (5 Points). Consider the differential equation for the driven harmonic oscillator:

$$\ddot{x}(t) + \gamma\dot{x}(t) + \omega_0^2 x(t) = F(t). \tag{5}$$

For a general driving force $F(t)$, with $F(t) = 0$ for $|t| > T$, write down an expression for the special solution.

— 15 Points —

— To be handed in prior to the tutorials on 12.06.2017 —