Humboldt–Universität zu Berlin
Institut für Physik
Prof. Dr. Kurt Busch

SoSe 2017

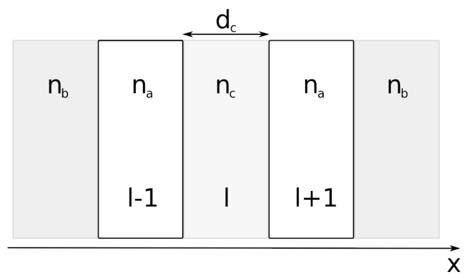
June 22nd, 2017

http://www.physik.hu-berlin.de/top/teaching

Computational Photonics: Problem Set 4

1 Scattering Matrix Method

In the following, we consider stacks of homogeneous layers with different refractive indices n_l that exhibit a periodicity along the x-direction (one-dimensional photonic crystal).



Use the recursive scattering matrix algorithm to build the photonic crystal with $n_a = 1$, $n_c = n_b = 2$. Also try adding a metallic layer ($n_c = 1+5i$) in the middle. Test the system for stability of the transmittance and reflectance calculations regarding the thickness dependence d_c of the metallic layer. In the following, we briefly outline the scattering matrix approach.

The difference of the scattering matrix to the transfer matrix is, that the S-matrix connects the incoming waves with the outgoing waves

$$\begin{pmatrix} A_l \\ B_0 \end{pmatrix} = S(0,l) \begin{pmatrix} A_0 \\ B_l \end{pmatrix}.$$

In order to propagate the wave one step further (either across an interface or free propagation over a distance) within the crystal, the transfer matrix T that corresponds to this step has to be connected with the scattering matrix S to form the new scattering matrix S'

$$\begin{aligned} S'_{11} &= T_{11}S_{11} - (T_{11}S_{12} + T_{12})(T_{21}S_{12} + T_{22})^{-1} T_{21}S_{11}, \\ S'_{12} &= (T_{11}S_{12} + T_{12})(T_{21}S_{12} + T_{22})^{-1}, \\ S'_{21} &= S_{21} - S_{22}(T_{21}S_{12} + T_{22})^{-1} T_{21}S_{11}, \\ S'_{22} &= S_{22}(T_{21}S_{12} + T_{22})^{-1}. \end{aligned}$$

Implement the transfer matrices first. The multiplication of two scattering matrices $S(0,N) = S^1(0,l) * S^2(l,N)$ is not as easy as in the transfer matrix problem

$$\begin{aligned} S_{11} &= S^2_{11}\left(1 - S^1_{12}S^2_{21}\right)^{-1} S^1_{11}, \\ S_{12} &= S^2_{11}\left(1 - S^1_{12}S^2_{21}\right)^{-1} S^1_{12}S^2_{22} + S^2_{12}, \\ S_{21} &= S^1_{21} + S^1_{22}S^2_{21}\left(1 - S^1_{12}S^2_{21}\right)^{-1} S^1_{11}, \\ S_{22} &= S^1_{22}\left(S^2_{21}\left(1 - S^1_{12}S^2_{21}\right)^{-1} S^1_{12} + 1\right) S^2_{22}. \end{aligned}$$

This is the price to pay for numerical stability!

— Discussion in the exercise on July 7th, 2017 —

Problems marked with * are voluntary. All others are **due** on **July 7th, 2017** at **8am** via e-mail to kurt.busch@physik.hu-berlin.de or have to be brought to the exercise class on a flash drive. Analytical solutions are accepted handwritten or in the .pdf format, numerical solution in a digital version.