HUMBOLDT–UNIVERSITÄT ZU BERLIN SoSe 2018
INSTITUT FÜR PHYSIK

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Fluctuation-induced Phenomena: Problem Set 12

1) Recovering Casimir's result (7 Points).

At zero temperature the Lifshitz' formula for the Casimir energy is

$$E_{\mathrm{Lif}}(L) = A\int_0^\infty \frac{\mathrm{d}\omega}{2\pi}\int \frac{\mathrm{d}^2\mathbf{k}}{(2\pi)^2}\sum_\sigma \hbar\,\mathrm{Im}\left[\ln\left(1 - r_1^\sigma(\omega,k) r_2^\sigma(\omega,k) e^{-2\kappa L}\right)\right]\ . \quad (1)$$

Similar to the Casimir-Polder energy, $E_{\mathrm{Lif}}(L)$ can be conveniently expressed as an integral over imaginary frequencies

$$E_{\mathrm{Lif}}(L) = A\int_0^\infty \frac{\mathrm{d}\xi}{2\pi}\int \frac{\mathrm{d}^2\mathbf{k}}{(2\pi)^2}\sum_\sigma \hbar\ln\left(1 - r_1^\sigma(\mathrm{i}\xi,k) r_2^\sigma(\mathrm{i}\xi,k) e^{-2\kappa L}\right), \quad (2)$$

where $k = |\mathbf{k}|$ and $\kappa = \sqrt{|\mathbf{k}|^2 + \xi^2/\mathrm{c}^2}$. One can recover Casimir's 1948 configuration involving perfectly reflecting surfaces by requiring that $r^{\mathrm{TM}} = 1 = -r^{\mathrm{TE}}$. In this case show that

$$E_{\mathrm{Lif}}(L) \xrightarrow[\text{limit}]{\text{perfect}} -\frac{\hbar c \pi^2 A}{720 L^3}. \quad (3)$$

Hint: it is useful to first change the $\mathbf{k}$*-integration into an integration over* κ*, and next consider the change of variables*

$$\int_0^\infty \int_{\xi/\mathrm{c}}^\infty \kappa\,\mathrm{d}\kappa\,\mathrm{d}\xi \equiv \int_0^\infty \kappa \int_0^{\mathrm{c}\kappa} \mathrm{d}\xi\mathrm{d}\kappa.$$

Also, it is useful to expand the logarithm in terms of the exponential function and integrate term by term.

— 7 Points —

— To be handed in prior to the tutorials on 17.07.2018 —