HUMBOLDT–UNIVERSITÄT ZU BERLIN
INSTITUT FÜR PHYSIK SoSe 2018

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# Fluctuation-induced Phenomena: Problem Set 4

**1) The central limit theorem** (4 Points). In physics, the Gaussian distribution is omnipresent and it plays a leading role in the theory of stochastic processes. Mathematically, this predominance can be partially understood through the central limit theorem. It says that, given a set of $n$ random variables $X_1, X_2, \ldots X_n$ each of them characterized by an independent probability distribution $P_i(x_i)$ (not necessarily identical) with zero average and the same finite variance $\sigma^2$, the distribution $P_n(z)$ of the random variable

$$Z = \frac{1}{\sqrt{n}} \sum_{i=1}^{n} X_i, \tag{1}$$

is Gaussian in the limit $n \to \infty$. The demonstration of this simple but rather profound result was only sketched in the script and relies on the use of the characteristic function. Repeat and complete the demonstration.

**2) Properties of the Ornstein-Uhlenbeck stochastic process*** (3 Points). The Ornstein-Uhlenbeck stochastic process is stationary, Markovian, and also Gaussian. One of the characteristic of this process is that the second-order correlation has the property

$$C_2(t_3 - t_1) = C_2(t_3 - t_2)C_2(t_2 - t_1). \tag{2}$$

Show that the only function that satisfies this property is an exponential.
*(Hint: A possible way it to get a differential equation for $C_2(\tau)$ using a Taylor expansion.)*

— 7 Points —
— To be handed in prior to the tutorials on 22.05.2018 —