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Fluctuation-induced Phenomena: Problem Set 6

- 1) **Driven harmonic oscillator** (5 Points). Consider the differential equation for the driven harmonic oscillator:

$$\ddot{x}(t) + \gamma\dot{x}(t) + \omega_0^2 x(t) = F(t). \quad (1)$$

For a general driving force $F(t)$, which is non-zero for a finite time interval only, write down an expression for the special solution.

- 2) **Independent oscillator model** (10 Points). Consider a system of coupled harmonic oscillators described by the Hamiltonian

$$\hat{H} = \frac{\hat{p}^2}{2M} + \frac{1}{2}M\omega_0^2\hat{x}^2 + \sum_{n=1} \frac{1}{2m_n} (\hat{p}_n^2 + m_n^2\omega_n^2[\hat{q}_n - \hat{x}]^2). \quad (2)$$

The oscillator with mass M and frequency ω_0 is our system, while the oscillators with mass m_n and frequency ω_n describe the environment.

- (a) Show that the previous Hamiltonian leads to the following equation of motions

$$\ddot{\hat{x}}(t) + \omega_0^2\hat{x}(t) = \sum_{n=1} \frac{m_n}{M}\omega_n^2[\hat{q}_n - \hat{x}], \quad \ddot{\hat{q}}_n(t) + \omega_n^2\hat{q}_n(t) = \omega_n^2\hat{x}(t) \quad (3)$$

- (b) Go in the Fourier space and show that combining the equations one obtains

$$\left[-\omega^2 + \omega_0^2 - \omega^2 \sum_{n=1} \frac{m_n}{M} \frac{\omega_n^2}{\omega_n^2 - \omega^2} \right] \hat{x}(\omega) = 0 \quad (4)$$

- (c) If we now consider the case where the frequencies ω_n become dense to form a continuum, one can write

$$-\omega^2 + \omega_0^2 - \omega^2 \sum_{n=1} \frac{m_n}{M} \frac{\omega_n^2}{\omega_n^2 - \omega^2} \rightarrow -\omega^2 + \omega_0^2 - i\omega\mu(\omega) \quad (5)$$

$$\text{with } \mu(\omega) = -i\omega \int_0^\infty d\nu \rho(\nu) \frac{m(\nu)}{M} \frac{\nu^2}{\nu^2 - \omega^2},$$

where $\rho(\nu)$ denotes the density of frequencies enabling us to transform the sum into an integral. Using $\rho(\nu) \frac{m(\nu)}{M} \nu^2 = \frac{2}{\pi}\gamma$, evaluate the integral in $\mu(\omega + i\epsilon)$ and consider $\epsilon \rightarrow 0$ at the end of the calculation. Discuss the implications of the sign of ϵ .

— 15 Points —

— To be handed in prior to the tutorials on 05.06.2018 —