HUMBOLDT–UNIVERSITÄT ZU BERLIN
INSTITUT FÜR PHYSIK

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Fluctuation-induced Phenomena: Problem Set 7

1) Thermal correlation (10 Points). One can use the independent oscillator model to derive a quantum version of the Langevin equation. The Langevin force $\hat{F}(t)$ has a symmetric correlation function given by

$$\frac{1}{2}\langle \hat{F}(t)\hat{F}(t') + \hat{F}(t')\hat{F}(t)\rangle = 2\frac{\gamma}{M}\int_{-\infty}^{\infty}\frac{\mathrm{d}\nu}{2\pi}u(\nu)e^{-\mathrm{i}\nu(t-t')} \ ,$$

where $u(\omega)$ is given by

$$u(\omega) = \frac{\hbar\omega}{2}\coth\left[\frac{\hbar\omega}{2k_BT}\right] \ .$$

Using the spectral representation of the hyperbolic cotangent,

$$\coth[x] = \sum_{n=-\infty}^{\infty}\frac{1}{x+\mathrm{i}n\pi} \ ,$$

and the residue theorem, show that if $\tau_{\mathrm{th}} = \hbar/[2\pi k_BT]$, then

$$\frac{1}{2}\langle \hat{F}(t)\hat{F}(t') + \hat{F}(t')\hat{F}(t)\rangle = \gamma\frac{k_BT}{M}\left(2\delta(\tau) - \frac{1}{2\tau_{\mathrm{th}}\sinh\left[\frac{\tau}{2\tau_{\mathrm{th}}}\right]^2}\right).$$

— 10 Points —

— To be handed in prior to the tutorials on 12.06.2018 —