HUMBOLDT–UNIVERSITÄT ZU BERLIN
INSTITUT FÜR PHYSIK

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Dr. Franceso Intravaia, Daniel Reiche

Fluctuation-induced Phenomena: Problem Set 8

1) The fluctuation-dissipation theorem (5 Points). The FDT can be demonstrated using the analytic properties within a stripe of the complex τ-plane of the correlation functions

$$C_{AB}(\tau) = \langle \hat{A}_0(\tau)\hat{B}_0\rangle = \mathrm{tr}\left[\hat{\mathrm{A}}_0(\tau)\hat{\mathrm{B}}_0\hat{\rho}_0\right] \text{ and}$$
$$C_{BA}(-\tau) = \langle \hat{B}_0(-\tau)\hat{A}_0\rangle = \mathrm{tr}\left[\hat{\mathrm{B}}_0(-\tau)\hat{\mathrm{A}}_0\hat{\rho}_0\right],$$

where $\hat{\rho}_0 = Z^{-1}e^{-\beta\hat{H}_0}$ is the thermal state ($\beta = [k_B T]^{-1}$ and $Z = \mathrm{tr}[e^{-\beta\hat{H}_0}]$) and

$$\hat{A}_0(t) = e^{\frac{\mathrm{i}}{\hbar}\hat{H}_0 t}\hat{A}e^{-\frac{\mathrm{i}}{\hbar}\hat{H}_0 t}, \quad \hat{B}_0(t) = e^{\frac{\mathrm{i}}{\hbar}\hat{H}_0 t}\hat{B}e^{-\frac{\mathrm{i}}{\hbar}\hat{H}_0 t}.$$

A more direct demonstration relies on the expansion of the correlation functions over the basis formed by the eigenvectors of the (unperturbed) Hamiltonian $\hat{H}_0|E\rangle = E|E\rangle$ and on the definition of the density of states $\rho(E)$ which allows us to write the closure relation

$$1 = \int dE\rho(E)|E\rangle\langle E|. \tag{1}$$

Using such an approach, show that

$$\int_{-\infty}^{\infty} d\tau\, e^{\mathrm{i}\omega\tau}C_{BA}(-\tau) = e^{-\beta\hbar\omega}\int_{-\infty}^{\infty} d\tau\, e^{\mathrm{i}\omega\tau}C_{AB}(\tau).$$

Hint: Carry out both Fourier transforms and show that they are indeed equal.

— 5 Points —
— To be handed in prior to the tutorials on 19.06.2018 —