

Online on the 02.07.2019

Discussion during the tutorial on the 05.07.2019

## Fluctuation-induced Phenomena: Problem Set 12

### 1) Recovering Casimir's result.

At zero temperature the Lifshitz' formula for the Casimir energy is

$$E_{\text{Lif}}(L) = A \int_0^\infty \frac{d\omega}{2\pi} \int \frac{d^2\mathbf{k}}{(2\pi)^2} \sum_\sigma \hbar \text{Im} \left[ \ln (1 - r_1^\sigma(\omega, k) r_2^\sigma(\omega, k) e^{-2\kappa L}) \right] . \quad (1)$$

Similar to the Casimir-Polder energy  $E_{\text{Lif}}(L)$  can be conveniently expressed as an integral over imaginary frequencies

$$E_{\text{Lif}}(L) = A \int_0^\infty \frac{d\xi}{2\pi} \int \frac{d^2\mathbf{k}}{(2\pi)^2} \sum_\sigma \hbar \ln (1 - r_1^\sigma(i\xi, k) r_2^\sigma(i\xi, k) e^{-2\kappa L}) , \quad (2)$$

where  $k = |\mathbf{k}|$  and  $\kappa = \sqrt{|\mathbf{k}|^2 + \xi^2/c^2}$ . One can recover Casimir's 1948 configuration involving perfectly reflecting surfaces by requiring that  $r^{\text{TM}} = 1 = -r^{\text{TE}}$ . In this case show that

$$E_{\text{Lif}}(L) \xrightarrow{\text{perfect limit}} -\frac{\hbar c \pi^2 A}{720 L^3} . \quad (3)$$

*Hint: it is useful to first change the  $\mathbf{k}$ -integration into an integration over  $\kappa$ , and next consider the change of variables*

$$\int_0^\infty \int_{\xi/c}^\infty \kappa \, d\kappa \, d\xi \equiv \int_0^\infty \kappa \int_0^{c\kappa} d\xi \, d\kappa .$$

*Also, it is useful to expand the logarithm in terms of the exponential function and integrate term by term.*